Constraint Semantics and its Application to Conditionals

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1. What is constraint semantics?
2. Why do I think we should develop it?
3. How does it work, in general?
4. How might it work for conditionals?

We can think of ordinary truth-conditional semantics as giving us constraints on cognitive states.

Speakers generally aim to get their addressees constrained to believe the proposition that truth-conditional semantics associates with their utterance.

But constraints on cognitive states can be more complicated than simply *believing a proposition.*

And *we communicate* more complicated constraints on cognitive states.

We also communicate constraints that seem to bear on affective and conative states.

For a given prima facie communicated constraint, truth-conditional semantics generally either

1. does not aim to deliver it, or
2. tries to cram it into the ‘belief in a proposition’ model.

Constraint semantics aims to deliver *whatever* constraints we communicate.

It doesn't matter whether they constrain us cognitively, affectively, or conatively.

And it doesn't matter whether the constraint involves just one proposition, or two or more propositions standing in some relation, or whatever.

Perhaps a ‘force-modifier semantics’ could deliver non-cognitive and/or non-propositional constraints.

But constraint semantics allows that 'constraint operators' can take open sentences. It can handle quantifiers that scope over (what it takes to be) constraint operators:

(1) Every inch of the floor might have paint on it.

Crucial questions:

1. How would a semantic interpretation function that delivers ‘constraints’ work?
2. To what extent should (and could) a constraint semantics be compositional?
3. Can constraint semantics give a better model for a fragment of natural language than truth-conditional semantics can?

How constraint semantics works

How constraint semantics works

Constraints are *sets of inadmissibles:*

- inadmissible credences in a proposition
- inadmissible credences / ratios for credences in a pair of propositions
- inadmissible selection functions / similarity orderings
- inadmissible preference orderings
- or whatever you like . . .

How constraint semantics works

The semantic value of a sentence is the characteristic function of a set of inadmissibles.

(In other words, the semantic value of a sentence is of type $\langle i, t \rangle$: a function from inadmissibles to truth-values.)

How constraint semantics works

A set of inadmissibles represents a constraint by representing the conditions that are, *as far as that constraint is concerned*, inadmissible.

How constraint semantics works

Semantic types of subsentential expressions:

Referring expressions	$\langle e \rangle$
Predicates	$\langle e, \langle i, t \rangle \rangle$
Quantifiers	$\langle \langle e, \langle i, t \rangle \rangle, \langle i, t \rangle \rangle$
Constraint operators	$\langle \langle i, t \rangle, \langle i, t \rangle \rangle$

('$\langle e, \langle i, t \rangle \rangle$': a function from entities (e) to functions from inadmissibles (i) to truth values (t).)

How constraint semantics works

So the semantic value of a predicate (e.g.) is a function from individuals to sets of inadmissibles (type $\langle e, \langle i, t \rangle \rangle$).

How constraint semantics works

I take the semantic value of a predicate to be a function from individuals to sets of inadmissible *credences.*

How constraint semantics works

Constraint operators change one kind of constraint into another.

For example, a constraint operator might take a set of inadmissible credences and change it into a set of inadmissible preference orderings.

How constraint semantics works

Consider

(2) John will rest.

The semantic value of the predicate 'will rest' is a function from an individual x to the set consisting of credences that are inadmissible given the belief that x will rest.

How constraint semantics works

This semantic value is combined (via functional application) with the semantic value of ‘John’ to yield (2)’s semantic value—to a first approximation, the set of credences that are inadmissible given the belief that John will rest.

How constraint semantics works

How are (2) and (3) related?

(3) John may rest.

How constraint semantics works

Given the logical form [may [John will rest]], ⟦may⟧ takes the semantic value of (2)—a *credal* constraint—and delivers the semantic value of (3)—a *normative* constraint.

More concretely (but making some unforced choices):

$[\![\text{John will rest}]\!]$ = the characteristic function of the set of all functions that take the proposition that John will rest to a value in $[0, 1)$ and are otherwise undefined.

More concretely (but making some unforced choices):

$[\![\text{may}]\!]$ takes sets of functions from propositions into values in $[0, 1]$ to sets of functions from propositions into $\{\textsc{impermissible}, \textsc{permissible}, \textsc{obligatory}\}$.

More concretely (but making some unforced choices):

In particular, $[\![\text{may}]\!]$ takes a constraint that makes credences in $[0, 1)$ to ϕ inadmissible, and returns a constraint that makes assignments of IMPERMISSIBLE to ϕ inadmissible.

More concretely (but making some unforced choices):

So $[\![\text{John may rest}]\!]$ = the characteristic function of the set consisting of the function that takes the proposition that John will rest to IMPERMISSIBLE and is otherwise undefined.

How compositional a theory do we need?

How compositional a theory do we need?

Some reasons to seek compositionality:

- Learnability
- Productivity
- The Frege-Geach point (or points)
- Embeddings (?) ...

How compositional a theory do we need?

We need at least enough compositionality to explain *sub-clausal* linguistic phenomena: in particular, modals scoped under quantifiers.

How compositional a theory do we need?

(4) Most of the applicants might be hired ...

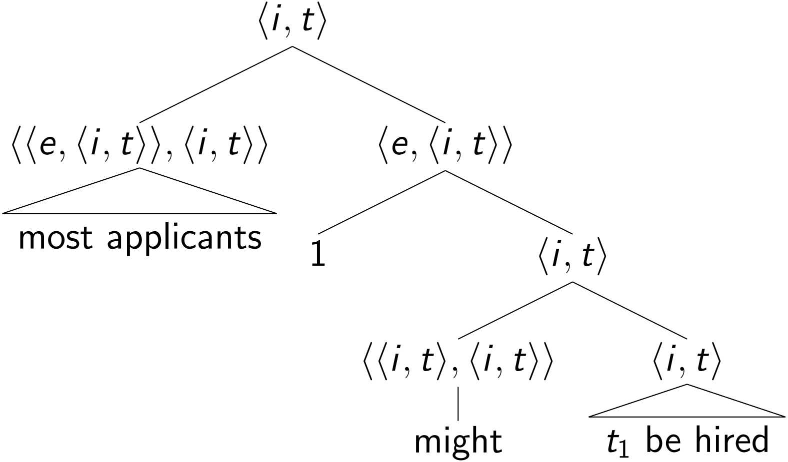
'For most of the applicants x, x might be hired.'
(MOST OF THE APPLICANTS $>$ MIGHT)

'It might be that most of the applicants are hired.'
(MIGHT $>$ MOST OF THE APPLICANTS)

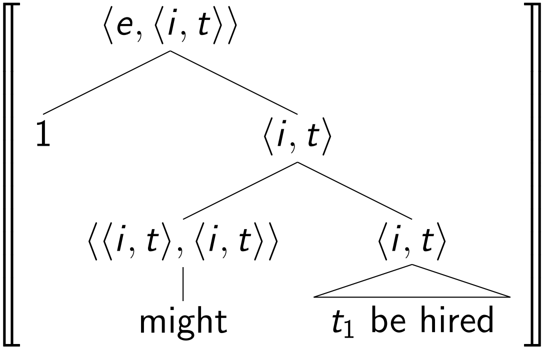
(5) ... but we will hire only one.

How compositional a theory do we need?

LF of (4), with wide-scope quantifier:



How compositional a theory do we need?



takes x to the set of credences that take the proposition that x is hired to values in $[0, \mu)$.

($\mu =$ the least credence sufficient for a 'might' belief)

How compositional a theory do we need?

Truth-conditional semantics treats quantifiers as properties of properties. We get TRUE iff the semantic value of the predicate has the quantifier's second order property.

How compositional a theory do we need?

In constraint semantics, a quantifier combines with a predicate whose semantic value is P to yield the same set of inadmissibles as

$$\bigvee_{X \in Q} \left(\bigwedge_{x \in X} P(x) \right)$$

(For ‘most of the applicants,’ $Q =$ the set of sets consisting of more than half of the applicants.)

How compositional a theory do we need?

$$[\![\text{most of the applicants . 1 (might } t_1 \text{ is hired)}]\!]$$

is the same set of inadmissibles as is yielded by

$$\bigvee_{X \in Q} \Big(\bigwedge_{x \in X} \lambda t_1(\text{might } t_1 \text{ is hired})(x) \Big)$$

How compositional a theory do we need?

Put another way:

$[\![$most of the applicants$]\!]$ is a function that takes an $\langle e, \langle i, t \rangle \rangle$ object (call it $P(\cdot)$) and yields the constraint corresponding to '$\big(P(\text{applicant 1}) \wedge P(\text{applicant 2}) \wedge \ldots\big) \vee \big(P(\text{applicant 2}) \wedge P(\text{applicant 3}) \wedge \ldots\big) \vee \ldots$'

How compositional a theory do we need?

So constraint semantics allows that

(4) Most of the applicants might be hired …

can be followed by

(5) … but we will hire only one.

without inconsistency.

How compositional a theory do we need?

Traditional force-modifier semantics simply aim to explain how propositions can be deployed with non-assertive force. But there's no proposition put forward with whatever force that provides the wide-scope quantifier meaning of (4).

How compositional a theory do we need?

The interaction between quantifiers and modals raises doubts about force-modifier approaches to modals. We need at least enough compositionality to explain how modals scope under quantifiers.

The application to conditionals

The application to conditionals

Conditionals can also scope under quantifiers:

(6) Most bubbles pop if touched.

This is *not* an assertion of ‘most bubbles pop’ conditional on their being touched.

The application to conditionals

Just as force-modifier accounts founder on

(4) Most of the applicants might be hired.

‘conditional assertion’ accounts founder on

(6) Most bubbles pop if touched.

The application to conditionals

Do linguistic phenomena like this force us to hold that indicative conditionals express propositions?

The application to conditionals

A simpler case, to start:

(7) If the bubble is touched, it will pop.

(7) expresses the constraint that one's probability that the bubble's popping conditional on its being touched is high; that one's cognitive state not be such that $Pr(C|A) \not\approx 1$.

The application to conditionals

So:

$$[\![\text{if } A,\ C]\!] = \{\text{credences making } Pr(C|A) \not\approx 1\}$$

The application to conditionals

Now, quantifying in:

$$[\![\text{most bubbles . 1 (if } x \text{ is touched, then } x \text{ pops)}]\!]$$

is the same set of inadmissibles as is yielded by

$$\bigvee_{X \in Q} \Big(\bigwedge_{x \in X} \lambda t_1 (\text{if } t_1 \text{ is touched, then } t_1 \text{ pops})(x) \Big)$$

The application to conditionals

So we have enough compositionality to explain how quantifiers can scope over modals and conditionals.

But we are not forced to say that all modalized statements and conditionals express propositions; they express constraints.

The application to conditionals

What about (8) and (9)?

(8) Most of the bubbles would have popped if they had been touched.

(9) If the bubble had been touched, it would have popped.

Do they demand a semantics radically different from that proposed for (6) and (7)?

The application to conditionals

Not necessarily.

We can make progress on this question by asking how the cognitive constraints associated with different kinds of conditionals are similar, and how they are different.

Thanks